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ASSESSMENT OF CENTERED DIFFERENCE SCHEMES ACCURACY FOR DYNAMIC PROBLEMS OF ELASTICITY THEORY IN INTERPOLATION SPACES

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Abstract. In the present paper, we investigate the accuracy of difference schemes for the first-order hyperbolic systems for the case of two-dimensional equations of dynamical theory of elasticity under weak smoothness assumptions on the solutions of the differential problem. Developing the apparatus of stability theory of difference schemes, we obtain an a priori error bound in a norm weaker than. Using this bound and the Bramble-Hilbert lemma, to estimate the approximation error, we prove $O(\tau^m + h^m)$ convergence of the scheme to the solution of the differential problem from the class $W_2^m(Q_T), m = 1, 2$. Besides, we obtained the accuracy of bounds in the interpolations space.

Keywords: equation dynamic elasticity theory, finite difference method, approximation, stability, convergence, interpolation space.

AMS Subject Classification: 65N06, 65N12.

1. Introduction

Study of convergence and accuracy of difference schemes takes the central place in the theory of numerical methods. Classic approach to study of convergence of difference schemes based on the Taylor formula sets high requirements to smoothness of solution (see the works [6], [7]). Therefore, in the recent time in the theory of difference schemes, increasing attention is paid to the issue of receiving the assessment of rate of convergence of difference schemes, at minimum requirements to smoothness of solving of a differential problem, i.e. receiving of concordant assessments of rate of convergence. For first time, such assessments had been received by A. A. Samarskiy, R.D. Lazarov and V. L. Makarov [7]. In particular, with help of operators of accurate difference schemes, assessments of rate of convergence of difference schemes for the following elliptical equations have been received, consistent with smoothness of desired solution,

$$\|y-u\|_{W_2^s(\omega)} \leq M\,|h|^{k-s}\,\|u\|_{W_2^k(\Omega)}\,,$$

where $|h|-$ characteristic size of grid, $0 \leq s \leq k$, s and $k-$ real numbers, $u(x)-$ solution of initial differential problem, $y(x)-$ solution of relevant difference scheme, $\|\cdot\|_{W_2^s(\omega)}$ and $\|\cdot\|_{W_2^k(\Omega)}$ $-$ Sobolev norms on set of functions of discrete and continuous argument, respectively, and $M-$ some constant not depending on the step of grid. The assessment of the below kind will be named as assessment of rate of convergence of difference schemes, concordant with smoothness of desired solution, for hyperbolic-type equation

$$E_{h,\tau}^{(s)}(t;z) \leq M(\tau^{k-s-1} + |h|^{k-s-1})\,\|u\|_{k,Q_T}\,, \tag{1}$$

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where $E^{(s)}_{h,\tau}(t;z)$ is some energy norm of s^{th} order similar to energy integral [2] as distinct from that, here we also consider the case $s<0$. Here $\|.\|_{k,Q_T}$ is norm in Sobolev space $W_2^k(Q_T), Q_T = \{(0,T)\times\Omega\}$, z is inaccuracy of numerical method.

Assessments in interpolation spaces are of great interest, i.e. assessments of type (1) with non-integral s and k. Such assessments are typical for finite-element method [8], [9].

In this work, such assessments will be received for difference schemes of approximating dynamical problems of elasticity theory.

2. Statement of problem

As it is known, many non-steady problems lead to solution of hyperbolic systems of first-order equations

$$D\frac{\partial U}{\partial t}=\sum_{k=1}^{p}A_k\frac{\partial U}{\partial x_k}+BU+F. \tag{2}$$

Here $x=(x_1,x_2,...,x_p)\in R_p$, $U(x,t)=(u_1,...,u_s,...u_m)(x,t)$, $F(x,t)=(f_1,...,f_s,...,f_m)(x,t)$ is function vector, D, A_k, B are real matrixes of degree $m\times m$. These matrixes in general case will depend on x and t. If matrixes D, A_k, B are symmetric, and matrix D is also a positive-definite matrix, then the system of equations (2) is named symmetric $t-$hyperbolic system [1].

Solution of the system (2) is sought in the field $\bar{Q}_T=\left\{x\in\bar{\Omega},\quad t\in[0,T]\right\}$,
$\bar{\Omega}=\left\{0\le x_\alpha\le l_\alpha, \alpha=\overline{1,p}\right\}$ is $p-$dimensional parallelepiped, and at $t=0$ it should satisfy the initial conditions

$$U(x,0)=U_0(x),\ x\in\bar{\Omega} \tag{3}$$

and on the boundary $\Gamma=\partial\bar{\Omega}$ of area $\bar{\Omega}$ to some boundary conditions.

In practice, solution of the system of first-order equations (2) with combined boundary conditions, i.e. problems with initial and boundary conditions are of the greatest interest.

From the general system of first-order equations (2) we may separate a special class of equations, which matrixes have a block-structure view

$$D=\begin{pmatrix} D_{11} & 0_{12}\\ 0_{21} & D_{22}\end{pmatrix},\quad A_k=\begin{pmatrix} 0^k_{11} & A^k_{12}\\ A^k_{21} & 0^k_{22}\end{pmatrix}, \tag{4}$$

where blocks $D_{11}, 0^k_{11}$ are matrixes of degree $s\times s$, $0_{12},\ A^k_{12}\ -\ s\times(m-s)$, $0_{21}, A^k_{21}-(m-s)\times s$, $D_{22},\ 0^k_{22}-(m-s)\times(m-s)$, in this case $0_{12}, 0_{21}, 0^k_{11}, 0^k_{22}$ are zero ones.

Choice of such classes of problems is explained by the fact that, first, many non-steady problems of mathematical physics lead to this very class of problems, for example, system of equations of acoustics, Maxwell, dynamic elasticity theory, magnetofluid dynamics, geomechanics, etc. Second, for such class of problems, in addition to normal schemes of first order of accuracy, we can build so-named centered difference schemes having second order of accuracy in time and in space.

Let's give definition of the notion of centered difference scheme. Let some differential equation

$$Lu=f,\quad L=\sum_{\alpha=1}^{P}L_\alpha$$

be approximated by the difference equation

$$L^h u=f^h,\quad L^h=\sum_{\alpha=1}^{P}L^h_\alpha. \tag{5}$$

Difference equation (5) will be named as centered, if templates S_α of operators L^h_α are symmetrical and centers of symmetry S_α coincide for all α.

An advantage of centered difference schemes is that in some cases they remove difficulties related to approximation of boundary conditions.

In this work, we consider system of equations of dynamic elasticity theory for velocities and stresses [1]

$$\begin{gathered}\frac{\partial\vartheta_\alpha}{\partial t}=\sum_{\beta=1}^{2}\frac{\partial\sigma_{\alpha\beta}}{\partial x_\beta}+f_\alpha,\\ \frac{\partial\sigma_{\alpha\beta}}{\partial t}=\delta_{\alpha\beta}(c_1^2-2c_2^2)\sum_{\gamma=1}^{2}\frac{\partial\vartheta_\gamma}{\partial x_\gamma}+c_2^2\left(\frac{\partial\vartheta_\alpha}{\partial x_\beta}+\frac{\partial\vartheta_\beta}{\partial x_\alpha}\right),\quad \alpha,\,\beta=1,2,\end{gathered}\tag{6}$$

where ϑ_α is components of vector of displacement speed; $\sigma_{\alpha\beta}=\sigma_{\beta\alpha}$ is components of stress tensor; f_α is components of vector of volumetric force; $c_1^2=(\lambda+2\mu)/\rho$, $c_2^2=\mu/\rho$ is velocities of longitudinal and transversal waves; ρ is density; λ,μ are Lame's constants; $\delta_{\alpha\beta}$ is Kronecker symbol, $\alpha,\beta=1,2$.

Solution of the system (6) is sought in the area $Q_T=\{x\in\bar\Omega,\ t\in[0,T]\}$, where $\bar\Omega=\{0\le x_\alpha\le l_\alpha,\quad \alpha=1,2\}$ is rectangle, with initial conditions

$$\vartheta_\alpha=\vartheta_\alpha^0(x),\quad \sigma_{\alpha\beta}=\sigma_{\alpha\beta}^0(x),\quad x\in\bar\Omega,\quad t=0\tag{7}$$

and boundary conditions

$$\begin{gathered}\vartheta_\alpha=\mu_1^{(-\alpha)},\sigma_{12}=\pi_2^{(-\alpha)},\quad x_\alpha=0,\\ \vartheta_\alpha=\mu_1^{(+\alpha)},\quad \sigma_{12}=\pi_2^{(+\alpha)},\quad x_\alpha=l_\alpha,\quad t\in(0,T],\quad \alpha=1,2.\end{gathered}\tag{8}$$

i.e. when normal component of displacements and tangent component of stress are set on the boundary of rectangle.

3. Difference schemes and assessment of accuracy

Let's define grids according to spatial variables

$$\bar\omega_\alpha=\left\{x_\alpha=i_\alpha h_\alpha,\ i_\alpha=\overline{0,N_\alpha}\right\},\quad \bar\omega_\alpha^*=\left\{\bar x_\alpha=(i_\alpha+0.5)\,h_\alpha,\quad i_\alpha=\overline{0,N_\alpha-1}\right\},$$

$$h_\alpha=l_\alpha/N_\alpha$$

and for time

$$\bar\omega_\tau=\left\{\ t_n=n\tau,\ n=\overline{0,M}\right\},\ \ \bar\omega_\tau^*=\left\{\ \bar t_n=(n+0.5)\tau,\ \ n=\overline{0,M-1}\right\},\ \ \tau=T/M.$$

Let grid functions y_α and $æ_{\alpha\beta}=æ_{\beta\alpha}$ approximate, respectively, ϑ_α and $\sigma_{\alpha\beta}$. Grid functions y_α and $æ_{\alpha\beta}$ will be determined on the grid

$$\begin{gathered}\bar\omega(y_1)=\bar\omega_1\times\bar\omega_2^*,\qquad \bar\omega(y_2)=\bar\omega_1^*\times\bar\omega_2,\ \bar\omega(æ_{11})=\bar\omega(æ_{22})=\bar\omega_1^*\times\bar\omega_2^*,\\ \bar\omega(æ_{12})=\bar\omega_1\times\bar\omega_2.\end{gathered}$$

Let's define the following averaging operators:

$$S^t u(x_1,x_2,t)=\frac{1}{\tau}\int\limits_{t_n}^{t_{n+1}}u(x_1,x_2,\ \eta)d\eta,\quad \overline{S}^t u(x_1,x_2,t)=\frac{1}{\tau}\int\limits_{\bar t-\tau}^{\bar t}u(x_1,x_2,\ \eta)d\eta,\ \ \bar t\ge\tau,$$

$$\overline{S}^t u(x_1,x_2,t)=\frac{2}{\tau}\int\limits_{0}^{0.5\tau}u(x_1,x_2,\ \eta)d\eta,\quad t=0,$$

$$S^{x_1}u=S^{x_1}u(\cdot,x_2,t)=\frac{1}{h_1}\int\limits_{x_{i_1}}^{x_{i_1+1}}u(\xi_1,x_2,t)d\xi_1$$

etc. If we apply the operators $S^t\bar{S}^{x_1}S^{x_2}$ and $S^tS^{x_1}\bar{S}^{x_2}$ sequentially to the first two equations (6), and apply $\overset{\wedge}{\bar{S}^t}\,S^{x_1}S^{x_2}$ to the third and forth equations, and finally $\overset{\wedge}{\bar{S}^t}\,\bar{S}^{x_1}\bar{S}^{x_2}$ to the last fifth equation, we will obtain the following integral correlations

$$\begin{aligned}
&(\bar{S}^{x_1}S^{x_2}\vartheta_1)_t = (S^tS^{x_2}\sigma_{11})_{\bar{x}_1} + (S^t\bar{S}^{x_1}\sigma_{12})_{x_2} + \varphi_1,\\
&(S^{x_1}\bar{S}^{x_2}\vartheta_2)_t = (S^t\bar{S}^{x_2}\sigma_{11})_{x_1} + (S^tS^{x_1}\sigma_{12})_{\bar{x}_2} + \varphi_2,\\
&(S^{x_1}S^{x_2}\bar{\sigma}_{11})_t = c_1^2(\overset{\wedge}{\bar{S}^t}\,S^{x_2}\vartheta_1)_{x_1} + (c_1^2-2c_2^2)\,(\overset{\wedge}{\bar{S}^t}\,S^{x_1}\vartheta_2)_{x_2},\\
&(S^{x_1}S^{x_2}\bar{\sigma}_{22})_t = (c_1^2-2c_2^2)\,(\overset{\wedge}{\bar{S}^t}\,S^{x_2}\vartheta_1)_{x_1} + c_2^2(\overset{\wedge}{\bar{S}^t}\,S^{x_1}\vartheta_2)_{x_2},\\
&(\bar{S}^{x_1}\bar{S}^{x_2}\bar{\sigma}_{12})_t = c_2^2\left[(\overset{\wedge}{\bar{S}^t}\,\bar{S}^{x_1}\vartheta_1)_{\bar{x}_2} + (\overset{\wedge}{\bar{S}^t}\,\bar{S}^{x_2}\vartheta_2)_{\bar{x}_1}\right],
\end{aligned}\tag{9}$$

where $\overset{\wedge}{\bar{S}^t} = \bar{S}^{t+\tau}$, $\varphi_1 = S^t\bar{S}^{x_1}S^{x_2}f_1$, $\varphi_2 = S^tS^{x_1}\bar{S}^{x_2}f_2$. For $t=0$ from the last three equations (1) we have

$$\begin{aligned}
S^{x_1}S^{x_2}\frac{\sigma_{11}(x,\bar{t}_0)-\sigma_{11}(x,0)}{0.5\tau} &= c_1^2(\bar{S}^tS^{x_2}\vartheta_1)^0_{x_1} + (c_1^2-2c_2^2)\,(\bar{S}^tS^{x_1}\vartheta_2)^0_{x_2},\\
S^{x_1}S^{x_2}\frac{\sigma_{22}(x,\bar{t}_0)-\sigma_{22}(x,0)}{0.5\tau} &= (c_1^2-2c_2^2)\,(\bar{S}^tS^{x_2}\vartheta_1)^0_{x_1} + c_2^2(\bar{S}^tS^{x_1}\vartheta_2)^0_{x_2},\\
\bar{S}^{x_1}\bar{S}^{x_2}\frac{\sigma_{12}(x,\bar{t}_0)-\sigma_{12}(x,0)}{0.5\tau} &= c_2^2\left[(\bar{S}^t\bar{S}^{x_1}\vartheta_1)^0_{\bar{x}_2} + (\bar{S}^t\bar{S}^{x_2}\vartheta_2)^0_{\bar{x}_1}\right].
\end{aligned}$$

Approximating integrals in (9)are defined by formula of average rectangles. We receive approximation of the system of equations (6)

$$\begin{aligned}
&y_{1,t} = \bar{æ}_{11,\bar{x}_1} + \bar{æ}_{12,x_2} + \varphi_1,\;\; y_{2,t} = \bar{æ}_{21,x_1} + \bar{æ}_{22,\bar{x}_2} + \varphi_2,\\
&\bar{æ}_{11,t} = c_1^2\hat{y}_{1,x_1} + (c_1^2-2c_2^2)\hat{y}_{2,x_2},\;\; \bar{æ}_{22,t} = (c_1^2-2c_2^2)\hat{y}_{1,x_1} + c_1^2\hat{y}_{2,x_2},\\
&\bar{æ}_{12,t} = c_2^2(\hat{y}_{1,\bar{x}_2} + \hat{y}_{2,\bar{x}_1}).
\end{aligned}\tag{10}$$

Here $y_1 = y_1(x_1,\bar{x}_2,t)$, $y_2 = y_2(\bar{x}_1,x_2,t)$, $\bar{æ}_{11} = æ_{11}(\bar{x}_1,\bar{x}_2,\bar{t})$, $\bar{æ}_{22} = æ_{22}(\bar{x}_1,\bar{x}_2,\bar{t})$, $\bar{æ}_{12} = æ_{12}(x_1,x_2,\bar{t})$. Other notations are taken from [6]. The system (10) will be added with initial conditions

$$\begin{aligned}
&y_1^0 = \bar{S}^{x_1}S^{x_2}\vartheta_1^0,\; y_2^0 = S^{x_1}\bar{S}^{x_2}\vartheta_2^0,\; æ_{11}^0 = S^{x_1}S^{x_2}\sigma_{11}^0,\; æ_{22}^0 = S^{x_1}S^{x_2}\sigma_{22}^0,\\
&æ_{12}^0 = \bar{S}^{x_1}\bar{S}^{x_2}\sigma_{12}^0,\quad \frac{\bar{æ}_{11}^0 - æ_{11}^0}{0.5\tau} = c_1^2y^0_{1,x_1} + (c_1^2-2c_2^2)y^0_{2,x_2},\\
&\frac{\bar{æ}_{22}^0 - æ_{22}^0}{0.5\tau} = (c_1^2-2c_2^2)y^0_{1,x_1} + c_1^2y^0_{2,x_2},\quad \frac{\bar{æ}_{12}^0 - æ_{12}^0}{0.5\tau} = c_2^2(y^0_{1,\bar{x}_2} + y^0_{2,\bar{x}_1})
\end{aligned}\tag{11}$$

and boundary conditions

$$\begin{aligned}
&y_\alpha = \bar{S}^tS^{x_{3-\alpha}}\mu_1^{(-\alpha)},\quad \bar{æ}_{12} = S^t\bar{S}^{x_{3-\alpha}}\pi_2^{(-\alpha)},\; x_\alpha = 0,\\
&y_\alpha = \bar{S}^tS^{x_{3-\alpha}}\mu_1^{(+\alpha)},\quad \bar{æ}_{12} = S^t\bar{S}^{x_{3-\alpha}}\pi_2^{(+\alpha)},\; x_\alpha = l_\alpha.
\end{aligned}\tag{12}$$

The system of difference equation (10) together with conditions (11), (12) presets centered difference scheme for problems (6) to (8). It refers to the class of schemes of running calculation: from the first two equations (10) we find $\hat{y}_\alpha$, and then from other equations we find $\hat{\bar{æ}}_{\alpha\beta}$ $\alpha,\beta = 1,2$.

Theorem 3.1. *Let solution of the problem (6)-(8) belong to space $W_2^k(Q_T)$ and the condition of stability of scheme (10)-(12) be fulfilled*

$$\gamma_1^2 + \gamma_2^2 \le c_\alpha^{-2}(1-\varepsilon)^2,\quad 0<\varepsilon<1,\quad \alpha = 1,2. \tag{13}$$

Then, solution of difference scheme (10)-(12) comes down in grid norm $\|\cdot\|_s$ *to solution of initial problem with velocity* $O(\tau^{k-s-1}+|h|^{k-s-1})$ *, with realization of assessment of accuracy*

$$\|Y\|_s \leq M(\tau^{k-s-1}+|h|^{k-s-1})\,\|U\|_{k,Q_T}\,,\quad 1<k-s\leq 3,\;\; s=-1,0,\;\; k=1,2,3, \tag{14}$$

where $Y=(y_1,y_2,\text{æ}_{11},\text{æ}_{22},\text{æ}_{12})$, $U=(\vartheta_1,\vartheta_2,\sigma_{11},\sigma_{22},\sigma_{12})$, $|h|^\alpha=h_1^\alpha+h_2^\alpha$,
$\|\cdot\|_{-1}=\|\cdot\|_{H^{-1}}\,,\;\|\cdot\|_0=\|\cdot\|_H\,.$

The proof of the theorem proving fully coincides with proving of theorem 3 and 4 of work [4]. Therefore we don't quote them herein. The definition of the spaces H and H^{-1} are given there as well.

4. Assessment of accuracy in interpolation spaces

On the basis of (14) we get assessment of accuracy in interpolation spaces [3].

Theorem 4.1. *Let's assume that* $U\in W_2^{\text{æ}}(Q_T)$ *and the condition of stability is satisfied (13). Then, solution of difference scheme (10)-(12) comes down in grid norm* H^θ *to accurate solution of initial problem, with realization of assessment of accuracy*

$$\|Z\|_\theta \leq M(\tau^{\text{æ}-\theta-1}+|h|^{\text{æ}-\theta-1})\,\|U\|_{\text{æ},Q_T}\,,\;\text{æ}\in[1,3],\;\theta\in[-1,0],\;2\leq\text{æ}-\theta\leq 3. \tag{15}$$

Proof. Let's introduce the spaces

$$\mathrm{H}_1=\left\{Z(x,t)\in C(\omega_\tau,W_2^2(\omega_h))\right\},\;\;\mathrm{H}_2=\left\{Z(x,t)\in C(\omega_\tau,W_2^1(\omega_h))\right\}.$$

We introduce space $\mathrm{H}_{\text{æ},2}$, $\text{æ}\in(1,2)$, interpolating Hilbert spaces $\mathrm{H}_1,\mathrm{H}_2$, and the norm $\forall Z\in\mathrm{H}_{\text{æ},2}$ is defined as follows

$$\|Z\|_{-1}=\left(\int_0^\infty K^2(Z,\alpha)\alpha^{-\text{æ}-1}d\alpha\right)^{1/2},\;K(Z,\alpha)=\inf_{V\in\mathrm{H}_1}\left\{\,\|Z-V\|_{\mathrm{H}_2}+\alpha\,\|V\|_{\mathrm{H}_1}\,,\,Z\in\mathrm{H}_2\right\}.$$

We introduce the operator R, which sets inaccuracy of scheme (10)-(12) in accordance $U(x,t)$ to solution of problem (6)-(8): $Z=Y-S^xU:\;RU=Z$. Then, it appears from (14) that norm of the operator $R:\;W_2^2(Q_T)\to\mathrm{H}_1$, is assessed as follows: $\|R\|_{-1}\leq M(\tau^2+|h|^2)$. In similar way, $R:\;W_2^1(Q_T)\to\mathrm{H}_2$ is assessed as:$\|R\|_{-1}\leq M(\tau+|h|)$. Then for $R:\;W_2^k(Q_T)\to\mathrm{H}_{\text{æ},2}$ with norm $\|R\|_{-1}\leq\|R\|_{\mathrm{H}_1}^{\text{æ}}\,\|R\|_{\mathrm{H}_2}^{1-\text{æ}}\leq M(\tau^{\text{æ}}+|h|^{\text{æ}})$ the following assessment is justified

$$\|Z\|_{-1}\leq M(\tau^{\text{æ}}+|h|^{\text{æ}})\,\|U\|_{\text{æ},Q_T}\,,\;\text{æ}\in(1,2). \tag{16}$$

Let's introduce spaces

$$\mathrm{H}_1^*=\left\{Z(x,t)\in C(\omega_\tau,W_2^3(\omega_h))\right\},\;\;\mathrm{H}_2^*=\left\{Z(x,t)\in C(\omega_\tau,W_2^2(\omega_h))\right\}.$$

We introduce space $\mathrm{H}_{\text{æ},2}^*$, $\text{æ}\in(2,3)$, interpolating Hilbert spaces $\mathrm{H}_1^*,\mathrm{H}_2^*$, and the norm $\forall Z\in\mathrm{H}_{\text{æ},2}^*$ is defined as follows

$$\|Z\|_0=\left(\int_0^\infty K^2(Z,\alpha)\alpha^{-\text{æ}+1}\frac{d\alpha}{\alpha}\right)^{1/2},\;K(Z,\alpha)=\inf_{V\in\mathrm{H}_1^*}\left\{\,\|Z-V\|_{\mathrm{H}_2^*}+\alpha\,\|V\|_{\mathrm{H}_1^*}\,,\;Z\in\mathrm{H}_2^*\right\}.$$

Then, it appears from (16) that norm of operator $R:\;W_2^3(Q_T)\to\mathrm{H}_1^*$, is assessed as follows: $\|R\|_0\leq M(\tau^2+|h|^2)$. In similar way, $R:\;W_2^2(Q_T)\to\mathrm{H}_2^*$ is assessed as: $\|R\|_0\leq M(\tau+|h|)$. Then for $R:\;W_2^k(Q_T)\to\mathrm{H}_{\text{æ},2}^*$ with norm $\|R\|_0\leq\|R\|_{\mathrm{H}_1^*}^{\text{æ}}\,\|R\|_{\mathrm{H}_2^*}^{1-\text{æ}}\leq M(\tau^{\text{æ}-1}+|h|^{\text{æ}-1})$ the following assessment is justified

$$\|Z\|_0\leq M(\tau^{\text{æ}-1}+|h|^{\text{æ}-1})\,\|U\|_{\text{æ},Q_T}\,,\;\text{æ}\in(2,3). \tag{17}$$

Let's introduce spaces

$$\mathrm{H}_1^{**}=\left\{Z(x,t)\in C(\omega_\tau,H^{-1}(\omega_h))\right\},\;\;\mathrm{H}_2^{**}=\{Z(x,t)\in C(\omega_\tau,L_2(\omega_h))\}.$$

We introduce space $\mathrm{H}_{\theta,\text{æ},2}$, $\theta \in (-1,0)$, interpolating Hilbert spaces $\mathrm{H}_1^{**}, \mathrm{H}_2^{**}$, and the norm $\forall Z \in \mathrm{H}_{\theta,\text{æ},2}$ is defined as follows

$$\|Z\|_\theta = \left(\int\limits_0^\infty K^2(Z,\alpha)\alpha^{-\text{æ}+\theta+1} \frac{d\alpha}{\alpha} \right)^{1/2}, \ K(Z,\alpha) = \inf_{V\in \mathrm{H}_1^{**}} \left\{ \|Z-V\|_0 + \alpha \|V\|_{(-1)}, Z \in \mathrm{H}_2^{**} \right\}.$$

Then, it appears from (16), (17) that norm of operator $R: \ W_2^{\text{æ}}(Q_T) \to \mathrm{H}_1^{**}$, is assessed as follows: $\|R\|_{(-1)} \le M(\tau^{\text{æ}} + |h|^{\text{æ}})$. In similar way, $R: \ W_2^{\text{æ}}(Q_T) \to \mathrm{H}_2^{**}$ is assessed as: $\|R\|_0 \le M(\tau^{\text{æ}-1} + |h|^{\text{æ}-1})$. Then for $R: \ W_2^{\text{æ}}(Q_\mathrm{T}) \to \mathrm{H}_{\theta,\text{æ},2}$ with norm $\|R\|_\theta \le \|R\|_{(-1)}^{\text{æ}} \|R\|_0^{1-\text{æ}} \le M(\tau^{\text{æ}-\theta-1} + |h|^{\text{æ}-\theta-1})$ the assessment is justified (15). The theorem is proved. □

5. Comment

The results of the theorem 2 are true for the firth boundary problem (see [4]), as well as for economical factorized centered difference scheme (the scheme is built and studied in the work [5]).

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